Fluid Flow Measurements

Flow Measuring Device

1. Pressure probes or pitot probes
2. Flow obstruction (volume flow rate)
   (a) Venturimeter
   (b) Orificemeter
   (c) Flow nozzles
3. Positive displacements
   (a) Rotary vane meter
   (b) Lobed impeller meter
   (c) Disc meter
4. Drag effects (volume flow rate)
   (a) Rotameter
   (b) Turbine meter
   (c) Vortex-sheding meter
   (d) Ultra-sonic meter
5. Anemeter type flow meter (velocity)
   (a) Hot wire
   (b) Hot-film
   (c) Laser doppler
   (d) Thermal mass
6. Magnetic flow meter
7. Flow visualisation
8. Shadow graph
9. Schliemann
10. Notches

Flow meters range widely in their level of **sophistication, size, cost, accuracy, versatility, capacity, pressure drop, and the operating principle**.

Some flowmeters **measure the flow rate directly** by discharging and recharging a measuring chamber of known volume continuously and keeping track of the number of discharges per unit time. But **most flowmeters measure the flow rate indirectly**—they measure the average velocity V or a quantity that is related to average velocity such as pressure and drag, and determine the volume flow rate from

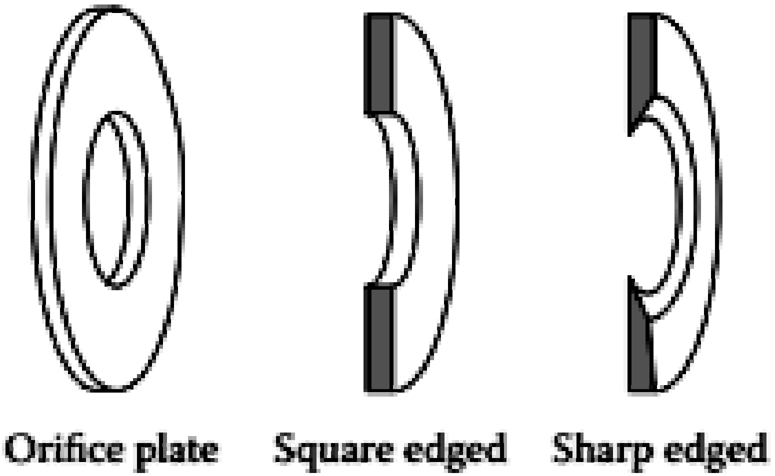
$$\dot{V} = VA_c$$

where A_c is the cross-sectional area of flow.

Obstruction Flowmeters:
Orifice, Venturi, and Nozzle Meters

Discharge measurement using **flow restrictions** **(for Internal Flows)** is widely used in industry. Restrictions include the orifice, nozzle and Venturi tube. **The idea is that the change in velocity leads to a change in pressure**. This Δp can be measured using a pressure gage (electronic or mechanical) or a manometer, and the flow rate inferred using either a theoretical analysis or an experimental correlation for the device. ***The flow rate is obtained by detecting the difference in pressures upstream and downstream of the device.***

The Orifice Plate



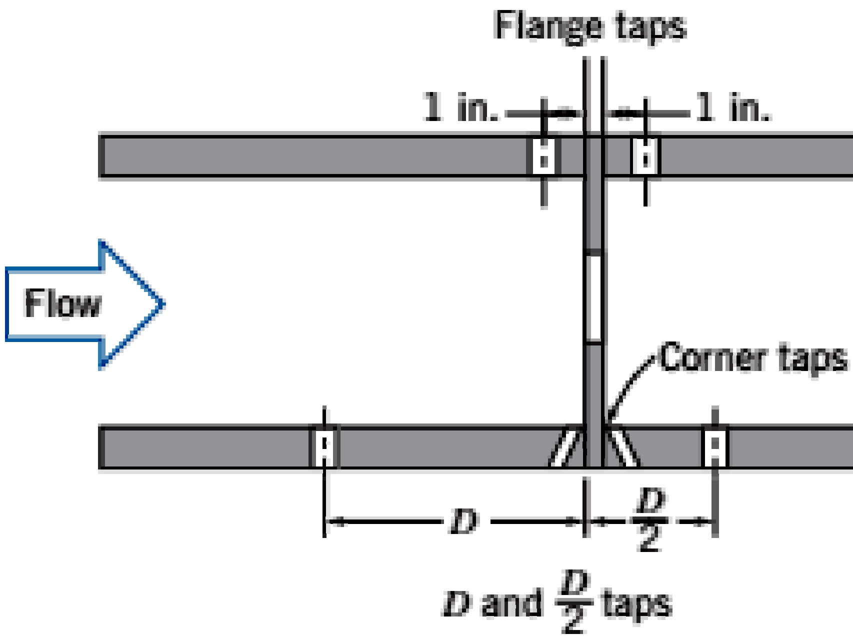
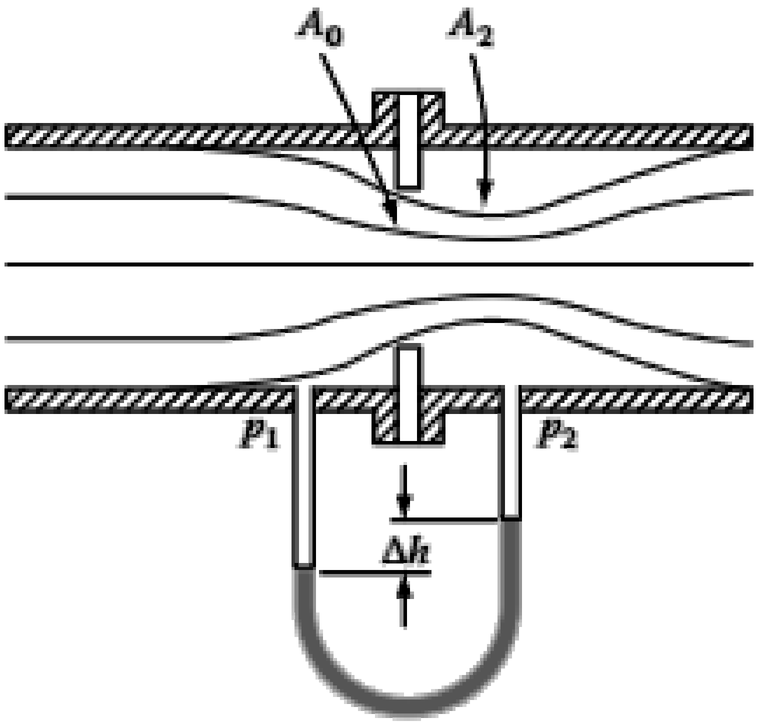
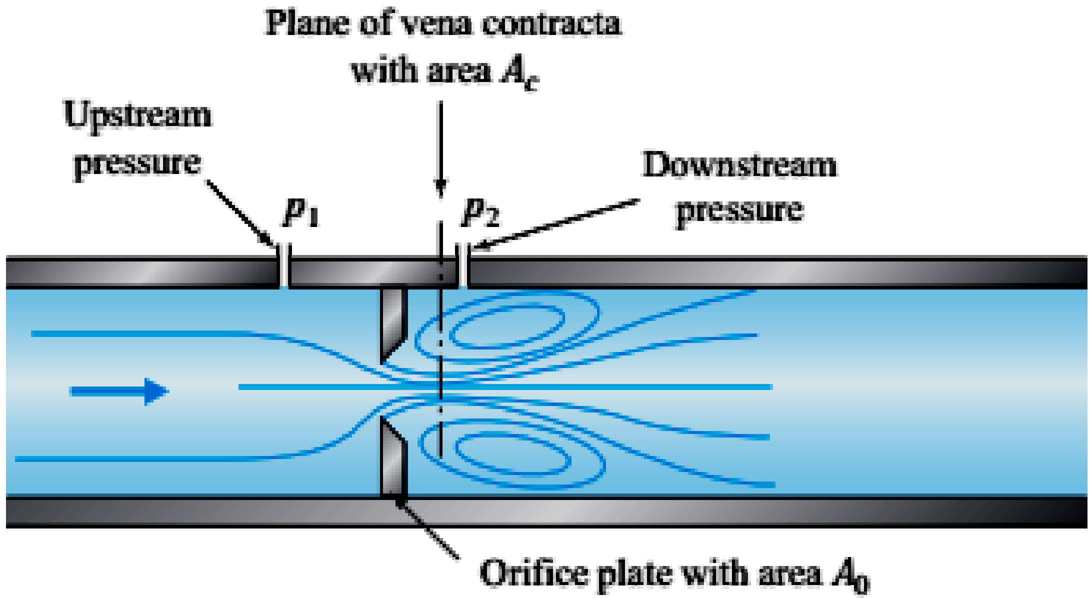


Fig1. Orifice geometry and pressure tap locations.





The orifice plate, merely a flat plate with a hole, is inserted into a pipeline, conventionally between flanges. **The hole can be either sharp edged or square edged**. As the flow goes through the plate, it follows a streamline pattern similar to that shown in Fig. Since its geometry is simple, ***it is low in cost and easy to install or replace***. ***The sharp edge of the orifice will not foul with scale or suspended matte***r. However, suspended matter can build up at the inlet side of a concentric orifice in a horizontal pipe; an eccentric orifice may be placed flush with the bottom of the pipe to avoid this difficulty. **The primary disadvantages of the orifice are** ***its limited capacity and the high permanent head loss caused by the uncontrolled expansion downstream from the metering element***. Downstream of the plate, the flow reaches a point of minimum area called a vena contracta.

Pressure taps are located at two positions: upstream of the restriction in the undisturbed flow region (location 1) and downstream at some location in the vicinity of the vena contracta (location 2) where the streamlines are uniform and parallel. The Bernoulli equation can be applied to points 1 & 2 to obtain an expression relating flow rate to pressure drop. Although the area at point 2 (the vena contracta) is unknown, it can be expressed in terms of the orifice area $A_{O.}$

$A_2 = C_C A_O$ where C_C is called a **contraction coefficient**.

Assumptions:
(1) Steady flow.
(2) Incompressible flow.
(3) Flow along a streamline.
(4) No friction.
(5) Uniform velocity at sections 1 and 2 .
(6) No streamline curvature at sections 1 or 2 , so pressure is uniform across those sections.
(7) $z_1 = z_2$

The Bernoulli equation:
(between upstream point 1 and vena contracta point 2)

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + \cancel{gz_1} = \frac{p_2}{\rho} + \frac{V_2^2}{2} + \cancel{gz_2}$$

$$p_1 - p_2 = \frac{\rho}{2}(V_2^2 - V_1^2) = \frac{\rho V_2^2}{2}\left[1 - \left(\frac{V_1}{V_2}\right)^2\right] \quad \text{--- (1a)}$$

and from **Continuity equation:**

$$(-\rho V_1 A_1) + (\rho V_2 A_2) = 0$$

$$V_1 A_1 = V_2 A_2 \quad \text{so} \quad \left(\frac{V_1}{V_2}\right)^2 = \left(\frac{A_2}{A_1}\right)^2 \quad \text{--- (1b)}$$

Substituting (1b) into (1a) gives, $$p_1 - p_2 = \frac{\rho V_2^2}{2}\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]$$

Solving for the ***theoretical velocity,*** V_2,

$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}} \quad \text{----- (1c)}$$

The ***theoretical mass flow rate*** is then given by

$$\dot{m}_{\text{theoretical}} = \rho V_2 A_2$$

$$= \rho \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}} A_2$$

Or,

$$\dot{m}_{\text{theoretical}} = \frac{A_2}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{2\rho(p_1 - p_2)} \quad \text{----- (1d)}$$

Equation 1(d) shows that, under the set of assumptions, for a given fluid (ρ) and flow meter geometry (A_1 and A_2), ***the flow rate is directly proportional to the square root of the pressure drop across the meter taps,***

$$\dot{m}_{\text{theoretical}} \propto \sqrt{\Delta p}$$

which is the basic idea of these devices.

Another form: (when $z_1 \neq z_2$, inclined pipe)

The Bernoulli equation: $$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

Consider, $h_1=\frac{p_1}{\gamma}+z_1$ and $h_2=\frac{p_2}{\gamma}+z_2$

So, $$h_1+\frac{V_1^2}{2g}=h_2+\frac{V_2^2}{2g}$$

$(h_1 - h_2)$ has been read from an attached manometer or pressure transducer.

$$V_2^2-V_1^2=2g(h_1-h_2)$$

$$V_2^2\left(1-\frac{V_1^2}{V_2^2}\right)=2g(h_1-h_2)$$

or, $$V_2=\sqrt{\frac{2g(h_1-h_2)}{1-\left({}^{V_1}/_{V_2}\right)^2}}$$

Continuity equation: $V_1A_1=V_2A_2$ so $\left(\frac{V_1}{V_2}\right)^2=\left(\frac{A_2}{A_1}\right)^2$

$$V_2=\sqrt{\frac{2g(h_1-h_2)}{1-(A_2/A_1)^2}}$$ ----- (2a)

$$Q_{theoretical}=A_2\sqrt{\frac{2g(h_1-h_2)}{1-(A_2/A_1)^2}}$$

$$\dot{m}_{theoratical}=\rho A_2\sqrt{\frac{2g(h_1-h_2)}{1-\left({}^{V_1}/_{V_2}\right)^2}}$$ ----- (2b)

The actual discharge differs from the ideal for two primary reasons.

Because of real fluid flow, friction causes the velocity at the centerline to be greater than the average velocity at each cross section. Second, ***the piezometric head h_2 evaluated at the vena contracta in the relation, is substituted with $h_{2/}$,*** the known reading at the downstream pressure tap. Also, ***since the area of the vena contracta is unknown, it is convenient in 2(b) to replace A_2 by C_cA_0,*** where C_c is the contraction coefficient and A_0 is the area of the orifice opening. Again, the relation between actual velocity and theoretical velocity at vena contracta is given by

$$C_V = \frac{actual\ velocity\ at\ vena\ contracta}{theoretical\ velocity\ at\ vena\ contracta}$$

where C_V is **the coefficient of velocity**

So, **the actual velocity at vena contracta** is given by

From equ. **1(c)**, $$V_{2a} = C_V\sqrt{\frac{2(p_1 - p_2)}{\rho[1-(A_2/A_1)^2]}}$$

$$V_{2a} = C_V\sqrt{\frac{2(p_1 - p_2)}{\rho[1-(C_cA_0/A_1)^2]}}$$ or, $$V_{2a} = C_V\sqrt{\frac{2(p_1 - p_2)}{\rho[1-C_c^2(D_0/D_1)^2]}}$$

Also from equ. 2**(a)**,

$$V_{2a} = C_V \sqrt{\frac{2g(h_1 - h_2)}{[1 - C_C^2 (D_0/D_1)^2]}}$$

So, **the actual flow rate** is given by

$$Q_a = A_2 \times V_{2a}$$

$$Q_a = C_C A_0 \times C_V \sqrt{\frac{2g(h_1 - h_2)}{[1 - C_C^2 (D_0/D_1)^2]}}$$

$$Q_a = C_d A_0 \sqrt{\frac{2g(h_1 - h_2)}{[1 - C_C^2 (D_0/D_1)^2]}}$$

where C_d is **the coefficient of discharge = C_V x C_C**

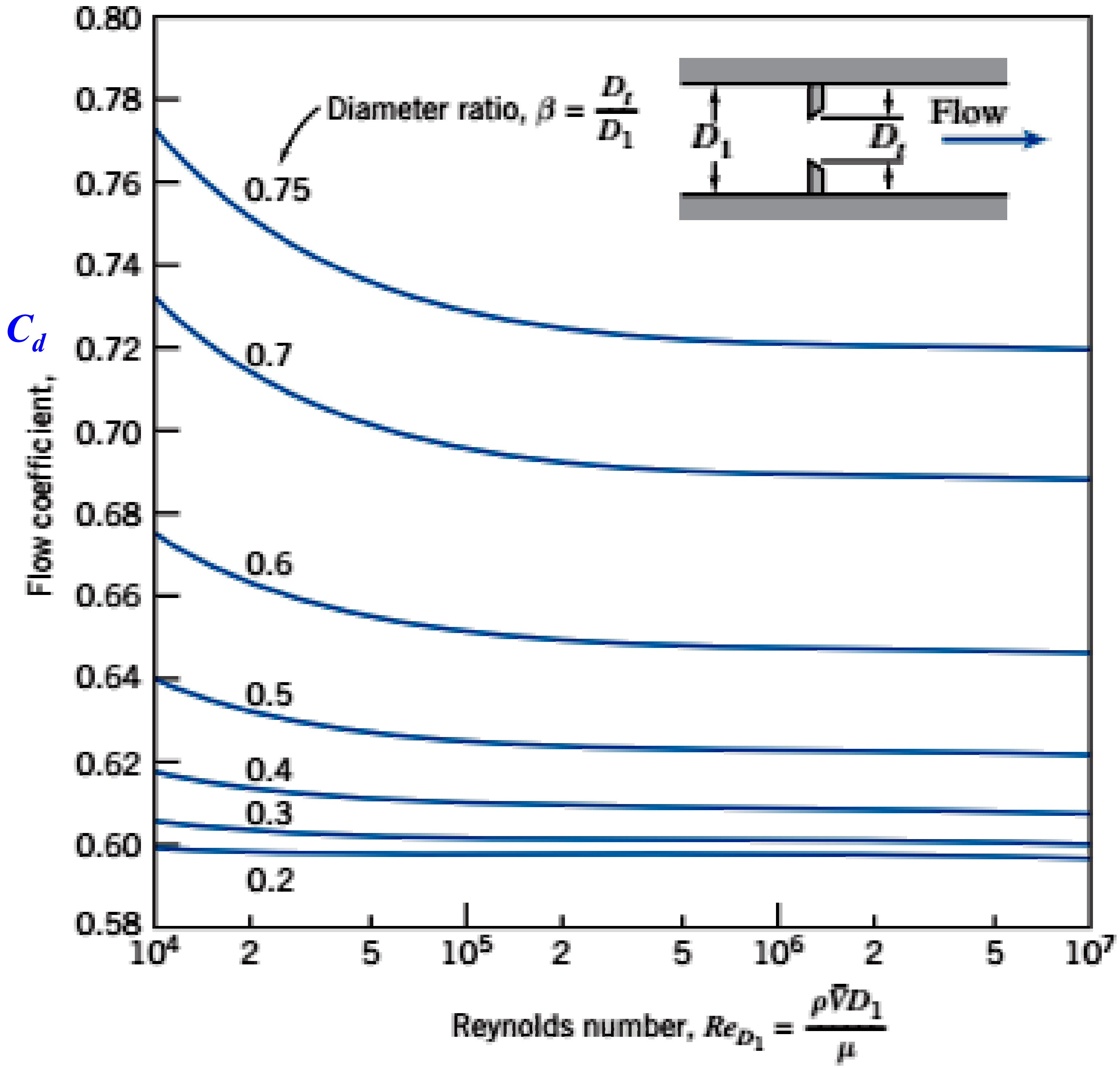
$$Q_a = C_d\, k \sqrt{(h_1 - h_2)}$$ Where $$k_1 = A_0 \sqrt{\frac{2g}{[1 - C_C^2 (D_0/D_1)^2]}}$$

$$Q_a = k_1 \sqrt{(h_1 - h_2)}$$ Where, $k_1 = C_d\, k$

Also from **(1)**,

$$Q_a = C_d A_0 \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - C_C^2 (D_0/D_1)^2]}}$$

The value of C_d depends on both β (ratio of orifice dia to pipe dia) and the Reynolds number Re $= V_1 D/\nu$.



C_d can be taken to be 0.61~ 0.65 for orifices.

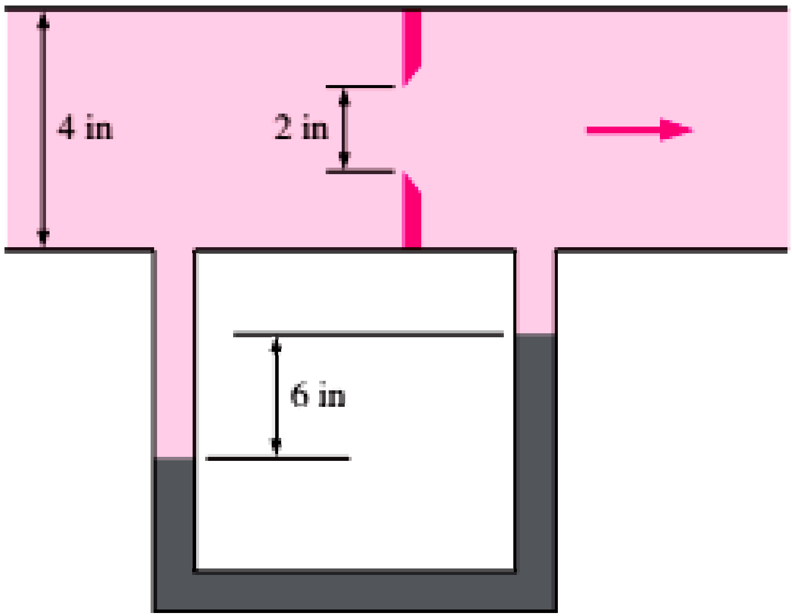
Fig: Flow coefficients for concentric orifices with corner taps.

$$\textit{Orifice meters:} \quad C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{\mathrm{Re}^{0.75}}$$

These relations are valid for $0.25 < \beta < 0.75$ and $10^4 < \mathrm{Re} < 10^7$. Precise values of C_d depend on the particular design of the obstruction, and thus the manufacturer's data should be consulted when available. ***For flows with high Reynolds numbers (Re 30,000), the value of C_d can be taken to be 0.61 for orifices.***

Problem:

An orifice with a 2-in-diameter opening is used to measure the mass flow rate of water at 60°F (ρ = 62.36 lbm/ft^3 and μ = 7.536 ×10^{-4} lbm/ft·s) through a horizontal 4-in-diameter pipe. A mercury manometer is used to measure the pressure difference across the orifice. If the differential height of the manometer is read to be 6 in, determine the volume flow rate of water through the pipe, the average velocity.



Solution:

Assumptions:

1. The flow is steady and incompressible.
2. The discharge coefficient of the orifice meter is $C_d = 0.61$.

Take the density of mercury to be 847 lbm/ft^3.

The diameter ratio and the throat area of the orifice are

$$\beta = d / D = 2/4 = 0.50$$

$$A_0 = \pi d^2 / 4 = \pi(2/12\text{ ft})^2 / 4 = 0.02182\text{ ft}^2$$

The pressure drop across the orifice plate can be expressed as

$$\Delta P = P_1 - P_2 = (\rho_{\text{Hg}} - \rho_{\text{f}})gh$$

Then the flow rate relation for obstruction meters becomes

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_{\text{Hg}} - \rho_{\text{f}})gh}{\rho_{\text{f}}(1-\beta^4)}} = A_o C_d \sqrt{\frac{2(\rho_{\text{Hg}} / \rho_{\text{f}} - 1)gh}{1-\beta^4}}$$

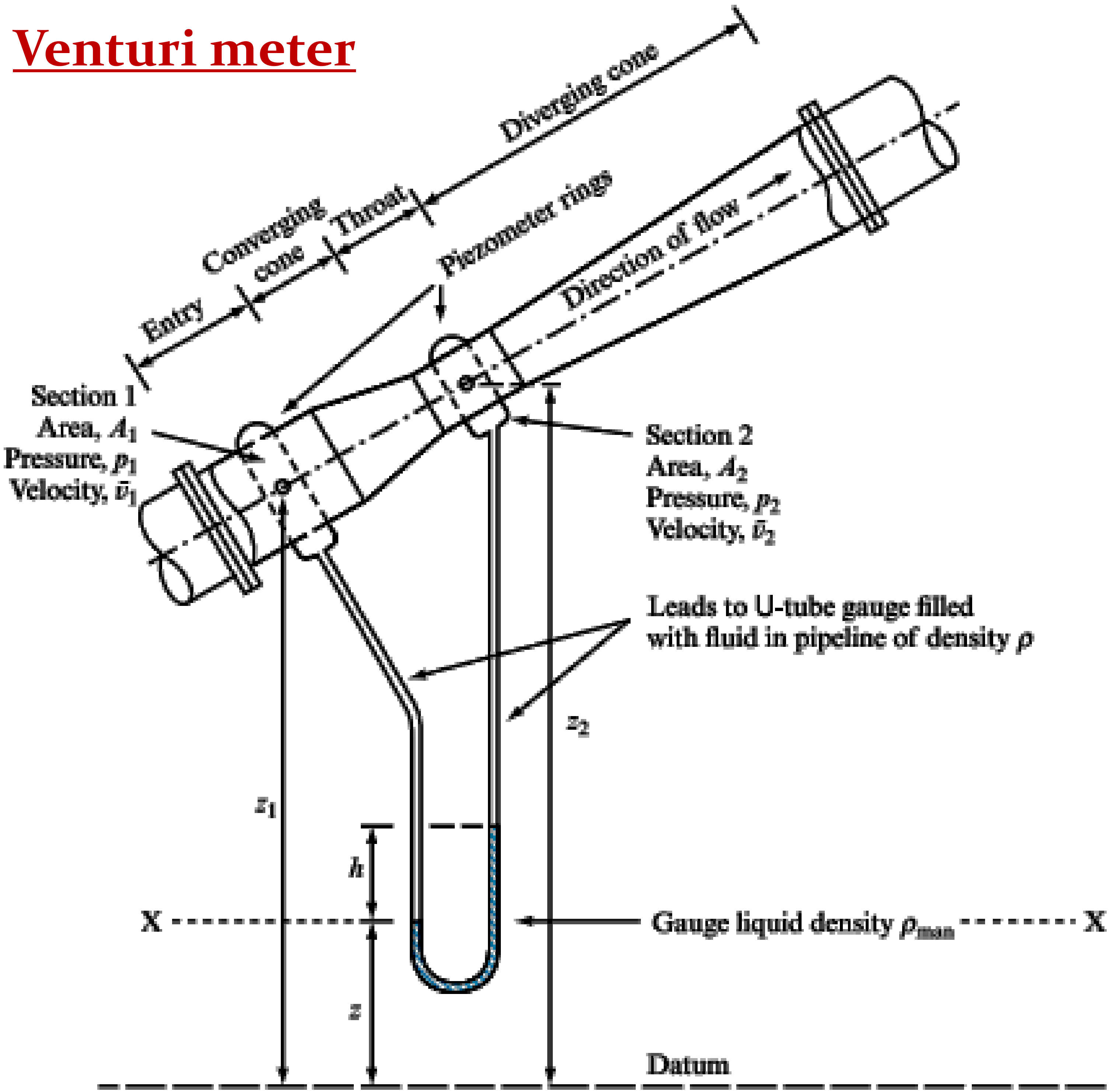
Substituting, the flow rate is determined to be

$$\dot{V} = (0.02182 \text{ ft}^2)(0.61)\sqrt{\frac{2(847/62.36-1)(32.2 \text{ ft/s}^2)(6/12 \text{ ft})}{1-0.50^4}} = \mathbf{0.277 \ ft^3/s}$$

The average velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe,

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.277 \text{ ft}^3/\text{s}}{\pi(4/12 \text{ ft})^2/4} = \mathbf{3.17 \ ft/s}$$

Venturi meter



A venturi meter consists of a short converging conical tube leading to a cylindrical portion, called the **throat**, of smaller diameter than that of the pipeline, which is followed by a diverging section in which the diameter increases again to that of the main pipeline. ***Its gradual contraction and expansion prevent flow separation and swirling and gives excellent pressure recovery***; ***it suffers only frictional losses on the inner wall surfaces; therefore, overall head loss is low***. ***Venturi meters are also self-cleaning because of their smooth internal contours***. The pressure difference from which the volume rate of flow can be determined is measured between the entry section 1 and the throat section 2, often by means of a U-tube manometer (as shown). The axis of the meter may be inclined at any angle. ***Venturi meters are heavy, bulky, and expensive***. Basically a casting lined with a corrosion-resistant material such as bronze, this device is placed directly in the flow line.

- **Assuming inviscid, and incompressible, steady flow and there is no loss of energy**, and the continuity equation between poimts1 and 2 are,

$$Q = A_1 V_1 = A_2 V_2 \quad \text{-------} \quad (3a)$$

Because the area $A_2 < A_1$, the continuity equation predicts that $V_1 < V_2$. In other words, the flow velocity must increase at the throat.

The Bernoulli equation for a frictionless flow through the meter is

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \qquad \text{-------} \quad (3b)$$

Because $V_1 < V_2$, then $p_1 > p_2$; that is, a pressure drop exists in the meter from upstream to the throat. Substituting from Equation (3b) for velocity, Equation (3b) becomes, after rearranging,

$$z_1 + p_1/\rho g + v_1^2/2g = z_2 + p_2/\rho g + v_2^2/2g,$$

$$v_2^2 - v_1^2 = 2g[(p_1 - p_2)/\rho g + (z_1 - z_2)]$$

For continuous flow,

$$\boxed{Q = A_1V_1 = A_2V_2} \qquad V_1 = \frac{A_2}{A_1}V_2$$

$$\frac{p_1 - p_2}{\rho g} + (z_1 - z_2) = \frac{V_2^2 - V_1^2}{2g}$$

$$2g\left[\frac{p_1 - p_2}{\rho g} + (z_1 - z_2)\right] = V_2^2 - \left(\frac{A_2}{A_1}V_2\right)^2$$

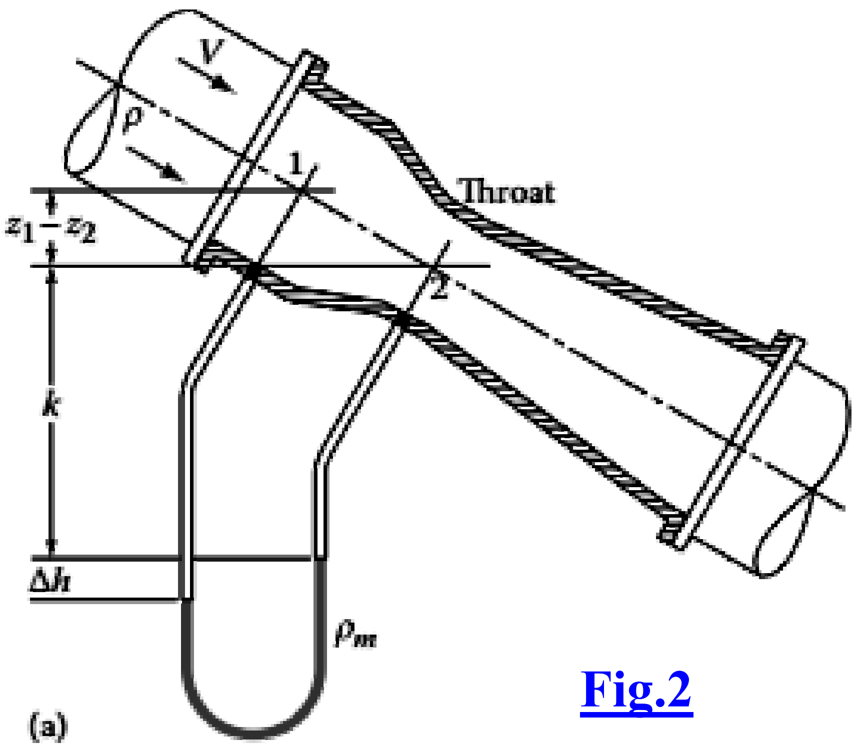


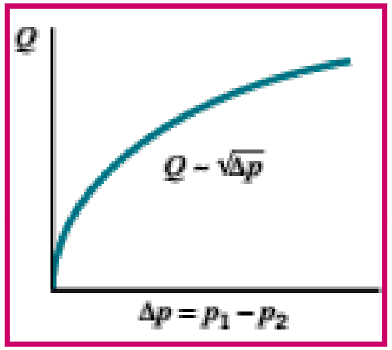
Fig.2

$$2g\left[\frac{p_1-p_2}{\rho g}+(z_1-z_2)\right]=V_2^2\left[1-\left(\frac{A_2}{A_1}\right)^2\right]$$

$$V_2=\sqrt{\frac{2g\left[\frac{p_1-p_2}{\rho g}+(z_1-z_2)\right]}{1-\left(\frac{A_2}{A_1}\right)^2}}$$ --- (3c)

The theoretical flow rate, $Q_{th} = A_2V_2$

$$Q=A_2\sqrt{\frac{2g\{[(p_1-p_2)/\rho g]+(z_1-z_2)\}}{1-(A_2^2/A_1^2)}}$$



Noting that $\frac{A_2}{A_1}=\frac{D_2^2}{D_1^2}$

$$Q_{th}=A_2\sqrt{\frac{2g\{[(p_1-p_2)/\rho g]+(z_1-z_2)\}}{1-(D_2^4/D_1^4)}}$$ ---- (3d)

The manometer reading provides the pressure drop required in the equation. From hydrostatics, we obtain the following for the manometer of Fig.2:

$$p_1+\rho g[(z_1-z_2)+k+\Delta h]=p_2+\rho gk+\rho_m g\Delta h$$

$$\frac{p_1-p_2}{\rho g}+z_1-z_2=\Delta h\frac{\rho_m-\rho}{\rho}=\Delta h\left(\frac{\rho_m}{\rho}-1\right)$$

Substitution into Equation (3d) yields

$$Q_{th}=A_2\sqrt{\frac{2g\Delta h(\rho_m/\rho-1)}{1-(D_2^4/D_1^4)}}$$ ---- (3e)

Thus, the theoretical flow rate through the meter is related to the manometer reading in such a manner that the meter orientation is not important; **the same equation results whether the meter is horizontal, inclined, or vertical**.

Due to frictional effects that are not accounted for in the Bernoulli equation, Q_{ac} is different from Q_{th}. The ratio of Q_{ac} to Q_{th} is called the venturi discharge coefficient, C_d:

$$C_d = \frac{Q_{ac}}{Q_{th}}$$

For each C_d that can be determined, a corresponding upstream Reynolds number can be calculated:

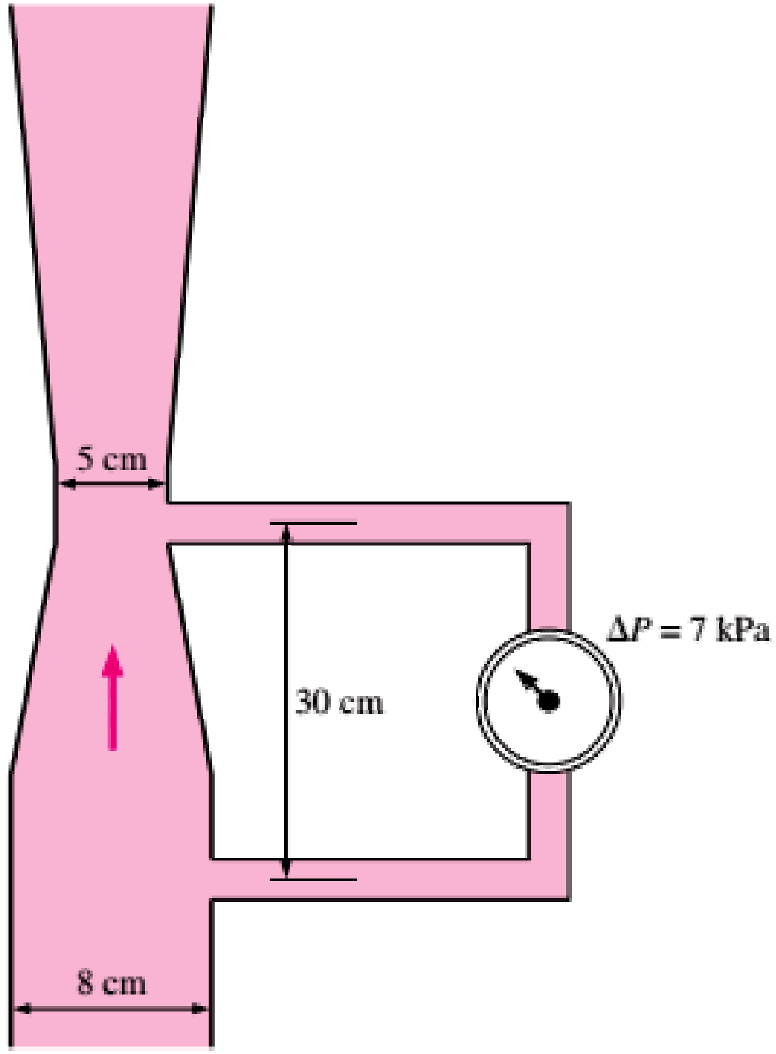
$$\mathrm{Re}_1 = \frac{V_1 D_1}{\nu} = \frac{4Q_{ac}}{\pi D_1^2}\frac{D_1}{\nu}$$

$$\mathrm{Re}_1 = \frac{4Q_{ac}}{\pi D_1 \nu}$$

Owing to its streamlined design, the discharge coefficients of ***Venturi meters are very high, ranging between 0.95 and 0.99*** (the higher values are for the higher Reynolds numbers) for most flows. In the absence of specific data, we can take **C_d = 0.98 for Venturi meters**.

Problem:

A vertical Venturi meter equipped with a differential pressure gage shown in Fig. is used to measure the flow rate of liquid propane at 10°C (ρ = 514.7 kg/m^3) through an 8-cm-diameter vertical pipe. For a discharge coefficient of 0.98, determine the volume flow rate of propane through the pipe.



Solution:

Assumptions: The flow is steady and incompressible.

Properties: The density of propane, ρ = 514.7 kg/m^3.
The discharge coefficient of Venturi meter, C_d = 0.98.

Analysis: The diameter ratio β and the throat area of the meter are

$$\beta = d / D = 5/8 = 0.625$$

$$A_0 = \pi d^2 / 4 = \pi (0.05\ \text{m})^2 / 4 = 0.001963\ \text{m}^2$$

Noting that ΔP = 7 kPa = 7000 N/m^2, the flow rate becomes

$$Q_{th} = A_2 \sqrt{\frac{2g\{[(p_1 - p_2)/\rho g] + (z_1 - z_2)\}}{1 - (D_2^4/D_1^4)}}$$

$$C_d = \frac{Q_{ac}}{Q_{th}}$$

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

$$= (0.001963\ \text{m}^2)(0.98)\sqrt{\frac{2 \times 7000\ \text{N/m}^2}{(514.7\ \text{kg/m}^3)((1-0.625^4)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)}$$

$$= \mathbf{0.0109\ m^3/s}$$

which is equivalent to 10.9 L/s. Also, the average flow velocity in the pipe is

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2 / 4} = \frac{0.0109\ \text{m}^3/\text{s}}{\pi (0.08\ \text{m})^2 / 4} = 2.17\ \text{m/s}$$

Problem:

A venturimeter of 150 mm × 75 mm size is used to measure the flow rate of oil having specific gravity of 0.9. The reading shown by the U tube manometer connected to the venturimeter is 150 mm of mercury column. Calculate the coefficient of discharge for the venturimeter if the flow rate is 1.7 m^3/min. (Note : The size of venturimeter generally specified in terms of inlet and throat diameters).

Solution:

Velocity $V_2 = \frac{C_d A_1}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh_m\left(\frac{\rho_m}{\rho} - 1\right)}$ and $Q = V_2 \times A_2$

Flow rate $Q = \frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh_m\left(\frac{\rho_m}{\rho} - 1\right)}$

Inlet area $A_1 = \frac{\pi}{4} \times 0.15^2 = 0.0177 \text{ m}^2,$

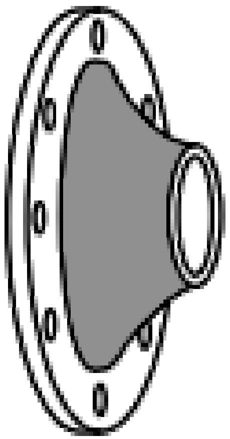
Throat area $A_2 = \frac{\pi}{4} \times 0.075^2 = 0.00442 \text{ m}^2$

Flow rate = (1.7/60) = 0.0283 m^3/s, Substituting

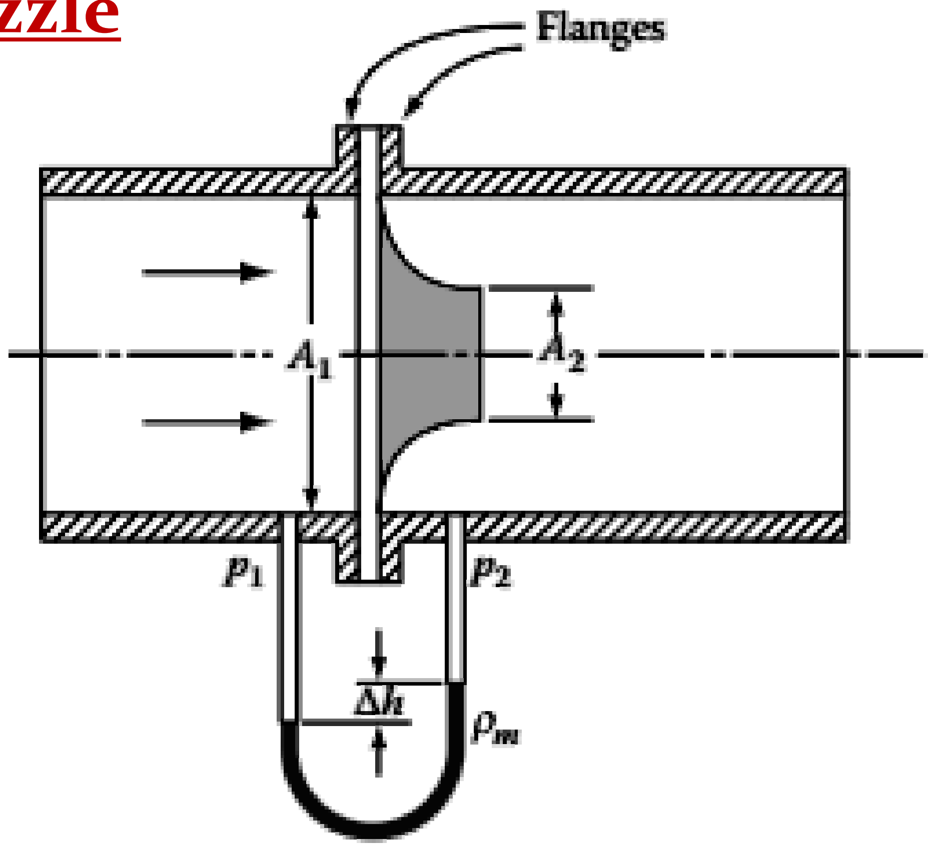
$$0.0283 = \frac{C_d \times 0.0177 \times 0.00442}{\sqrt{0.0177^2 - 0.00442^2}} \sqrt{2 \times 9.81 \times 0.15\left(\frac{13.6}{0.9} - 1\right)}$$

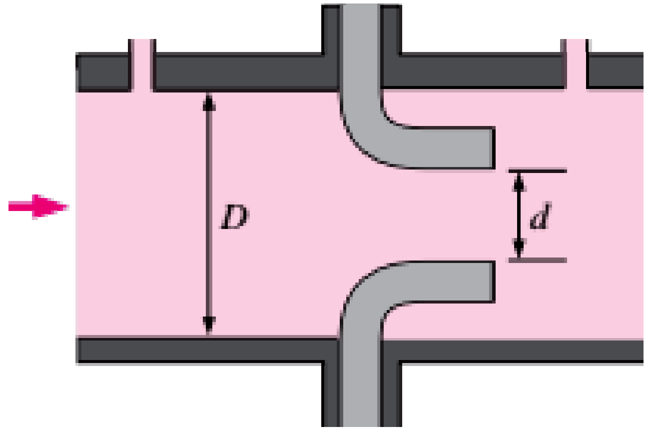
$$C_d = 0.963$$

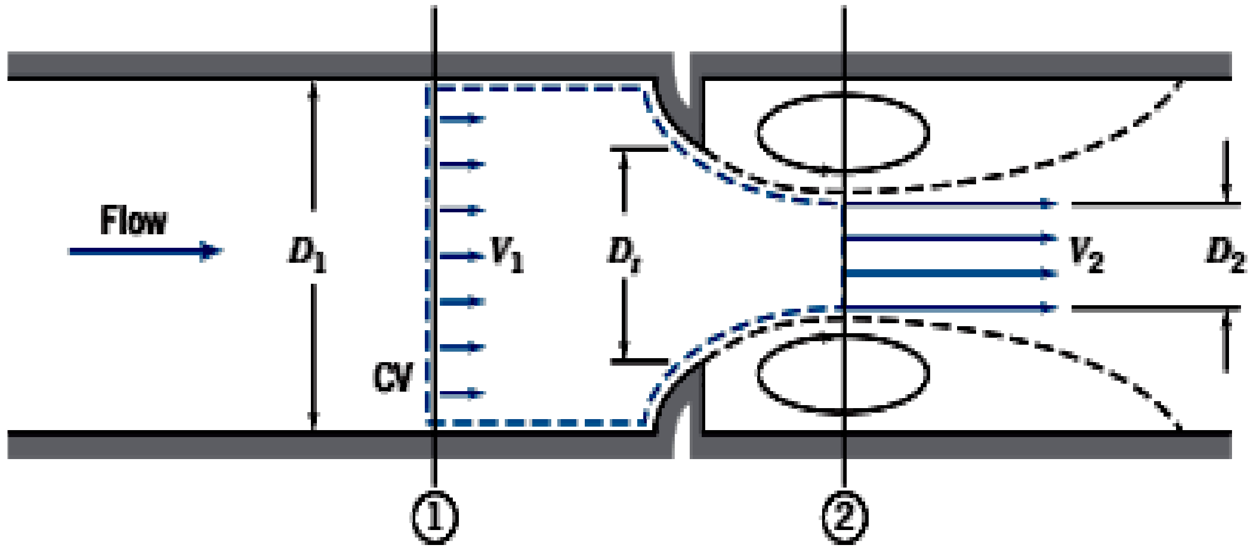
Flow nozzle



Flow nozzle







It consists of a standardized shape with pressure taps typically located one diameter upstream of the inlet and one-half diameter downstream. There are two standard shapes, either the long radius or the short radius. Due to the lack of an expansion section downstream of the nozzle, ***the total head loss is similar to that of an orifice, except that the vena contracta is nearly eliminated*** and ***the discharge coefficient is nearly unity***. ***The flow nozzle has an advantage over the orifice plate in that it is less susceptible to erosion and wear, and relative to the venturi meter, it is less expensive and simpler to install.***

As liquid passes through, a region of flow separation and reversal exists just downstream, and this adds to the losses encountered in a flow nozzle. The Bernoulli equation can be applied to the flow nozzle as was done for the venturi. The results are identical for the theoretical flow rate. For the flow nozzle, then, we get

$$Q_{th} = A_2\sqrt{\frac{2g(p_1 - p_2)}{\rho g(1 - D_2^4/D_1^4)}} = A_2\sqrt{\frac{2g\Delta h(\rho_m/\rho - 1)}{1 - (D_2^4/D_1^4)}}$$

Above equation gives the theoretical flow rate through the meter. Owing to losses, however, the actual flow rate is less. We therefore introduce a discharge coefficient for the nozzle defined as

$$C_d = \frac{Q_{ac}}{Q_{th}}$$

For flows with high Reynolds numbers (Re 30,000), the value of C_d can be taken to be 0.96 for flow nozzles

Nozzle meters:

$$C_d = 0.9975 - \frac{6.53\beta^{0.5}}{\text{Re}^{0.5}}$$

These relations are valid for $0.25 < \beta < 0.75$ and $10^4 < \text{Re} < 10^7$. Precise values of C_d depend on the particular design of the obstruction, and thus the manufacturer's data should be consulted when available. For flows with high Reynolds numbers (Re 30,000), ***the value of C_d can be taken to be 0.96 for flow nozzles***.

Owing to its streamlined design, the discharge coefficients of Venturi meters are very high, ranging between 0.95 and 0.99 (the higher values are for the higher Reynolds numbers) for most flows. In the absence of specific data, we can take $C_d = 0.98$ for Venturi meters.